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J. N. RICE, *On the in-and-circumscribed triangles of the plane rational quartic curve*, Washington, D. C., 1917. 32 pp. (Catholic University of America, 1917.)

P. R. RIDER, "The space problem of the calculus of variations in terms of angle." Pp. 241-256. [Reprinted from *American Journal of Mathematics*, 1917.] (Yale University, 1915.)

J. F. RITT, "On a general class of linear homogeneous differential equations of infinite order with constant coefficients, preceded by a note on the resolution into partial fractions of the reciprocal of an entire function of genus zero." Pp. 21-49. [Reprinted from *Transactions of the American Mathematical Society*, 1917.] (Columbia University, 1917.)

A. J. SCARLETT, *The surface tension of mixed liquids*, Easton, Pa., Eschenbach Printing Company, 1917. 34 pp. (Columbia University, 1917.)

C. P. SOUSLEY, "Invariants and covariants of the Cremona cubic surface." Pp. 135-146. [Reprinted from *American Journal of Mathematics*, 1917.] (Johns Hopkins University, 1915.)

J. M. STETSON, "Conjugate systems of curves on a surface both of whose Laplace transforms are lines of curvature." Pp. 106-126. [Reprinted from *Annals of Mathematics*, 1917.] (Princeton University, 1914.)

DOROTHY STIMSON, *The gradual acceptance of the Copernican theory of the universe*, Hanover, N. H., 1917. 147 pp. (Columbia University, 1917.)

H. G. TYLER, *Equations for imperfect gases*, Boston, F. H. Gilson Company, 1917. 5 + 30 pp. (New York University, 1916.)

J. L. WALSH, On the location of the roots of the Jacobian of two binary forms. Published as: (a) "On the location of the roots of the Jacobian of two binary forms, and of the derivative of a rational function," *Transactions of the American Mathematical Society*, vol. 19, 1918, 291-298; vol. 22, 1921, 101-116; (b) "On the location of the roots of the derivative of a polynomial," *Annals of Mathematics*, second series, vol. 22, 128-144, 1920. (Harvard, 1920.)

W. H. WILSON, "On a certain general class of functional equations." 20 pp. [Reprinted from *American Journal of Mathematics*, 1918.] (University of Illinois, 1917.)

PROBLEMS AND SOLUTIONS.

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Send all communications about Problems and Solutions to **B. F. FINKEL**, Springfield, Mo.

PROBLEMS FOR SOLUTION.

[N. B. Problems containing results believed to be new, or extensions of old results are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known text-books, or results found in readily accessible sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist the members of the Association with their difficulties in the solution of such problems.]

2928.

Show that if through the end P (opposite from the origin) of the loop of (a) the folium of Descartes, $x^3 + y^3 = 3axy$, (b) the strophoid or logocyclic curve, $x(x^2 + y^2) + a(x^2 - y^2) = 0$, a straight line be drawn meeting the curve again in Q and R , then QR always subtends a right angle at the origin (compare 1916, 90-92; also Basset, *Treatise on Cubic and Quartic Curves*, Cambridge, 1901, p. 82). Are these results particular cases of a general result for a certain class of cubic curves?

2929. Proposed by R. E. GAINES, University of Richmond.

Denote by A , O and B , respectively, the points $(-1, 0)$, $(0, 0)$ and $(1, 0)$ on the curve $y^2 = x^3 - x$, and let P be a variable point on the curve. Let PA and PO meet the curve again in Q and R , respectively, and let BQ and BP meet AR in M and N , respectively. Prove that QN is perpendicular to PM .

2930. Proposed by R. E. GAINES, University of Richmond.

If in reducing p/q (p and q integers, $q > p$) to a decimal the remainder $q - p$ ever appears, then the fraction will give a repeating decimal the number of digits in whose repetend will be exactly twice the number of digits in the quotient already obtained, and the remaining digits may, without further division, be obtained by subtracting the quotient already found from a succession of 9's.

2931. Proposed by R. C. ARCHIBALD, Brown University.

From the equality $\sec(\pi/14) + \sec(3\pi/14) - \sec(5\pi/14) = 0$ find a relation between the lengths of the side and diagonals of a regular inscribed heptagon.

2932. Proposed by R. C. ARCHIBALD, Brown University.

De Ville gave in 1629 the following construction for an approximation to the side of a regular polygon of n sides inscribed in a given circle: On a diameter AB construct an equilateral triangle ABC ; divide AB into n equal parts, join C to D , the first point of division of AB , and let CD produced intersect the circumference in E ; then the chord of the arc AF , the double of AE , will be approximately equal to the required side of the polygon. Construct a table of values for $n = 5$ to $n = 20$ exhibiting the extent, in minutes and seconds, to which the above construction is in error in connection with the central angle subtended by the constructed side of the polygon. [Somewhat similar tables have been made for constructions by Bosse and Bernard.]

2933.

Dudeney's Problem, 1902: With ruler and compasses only, divide an equilateral triangle into four rectilinear pieces which may be put together so as to form a square.

2934. Proposed by NATHAN ALTSHILLER-COURT, University of Oklahoma.

The base of a variable triangle is fixed, the opposite vertex describing a given straight line. Find the locus of the center of the nine-point circle and the envelope of the Euler line. [Editorial Note—In his inaugural dissertation at the University of Bern, Otto Jugi discussed (*Ueber den Feuerbach'schen Kreis in variablen Dreiecken*, Langenthal, 1903) the locus of the center of this nine-point circle when the vertex of the triangle moved (a) along any straight line, and in particular when this line is parallel to the base; (b) along a circle, an ellipse, a hyperbola, a parabola; (c) along a cubic parabola, Neil's parabola, a cissoid, a strophoid; and (d) along a lemniscate. In a number of cases the envelope of the nine-point circle was also found.]

2935.

Find the integral solutions of the equation $x! + 1 = y^2$.

[Proposed in *Gazeta Matematică*, April, 1921.]

2936. Proposed by J. P. BALLANTINE, University of Michigan.

A person in drinking from a conical drinking glass, tips it at a constant angular rate. At what angle will the delivery be the maximum and at what angle will the surface of the water be a maximum.

2937. Proposed by C. F. GUMMER, Queen's University.

A straight uniform stone wall AB is to be rebuilt in another position CD , the intersection of AB and CD being interior to CD but not to AB . How should the material be moved for the average horizontal distance through which it is carried to be as short as possible? If CD passes through B , show that the lower limit to the average distance may be expressed by means of logarithms and algebraic functions.

2938. Proposed by C. F. GUMMER, Queen's University.

If $a, b, \dots, i$ are real numbers ≥ 0 , and if $\begin{vmatrix} a^r & b^r & c^r \\ d^r & e^r & f^r \\ g^r & h^r & i^r \end{vmatrix}$ is equal to zero for five real values

of r other than zero, prove that the determinant $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ has either two rows or two columns proportional, or a single row or column of zeros.

2939. Proposed by C. F. GUMMER, Queen's University.

Show that the determinant

$$\begin{vmatrix} a_1 & a_2 & \cdots & a_n & -b_1 & 0 & \cdots & 0 \\ 0 & a_1 & \cdots & a_{n-1} & -b_2 & -b_1 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & a_1 & -b_n & -b_{n-1} & \cdots & -b_1 \\ b_1 & b_2 & \cdots & b_n & a_1 & 0 & \cdots & 0 \\ 0 & b_1 & \cdots & b_{n-1} & a_2 & a_1 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & b_1 & a_n & a_{n-1} & \cdots & a_1 \end{vmatrix}$$

has the property that every n -rowed determinant of the n left-hand columns is equal to its cofactor.

2940. Proposed by R. E. GILMAN, Brown University.

Find the average number of operations required to obtain m white balls from an urn containing p white and q black balls ($m < p$) as follows: For the first operation m balls are drawn simultaneously and examined, and such as are found to be black, are returned to the urn. Each subsequent operation is like the first save that the number of balls drawn is equal to the number replaced in the urn in the preceding operation.

NOTES

24. Problems discussed by Huygens.—In the last published volume (14, 1920) of *Oeuvres Complètes de Christian Huygens* (cf. 1921, 79) many problems are discussed. Among them are the following four: (a) [pages 208–209] dated 1655, “Given the sum of the sides of a right angled triangle and the difference of the segments of the hypotenuse made by a perpendicular dropped from the right angle, to construct the triangle”; (b) [pages 271–272] dated 1657, “Given two circles and a straight line, to describe a circle tangent to the given circles and with an arc cut off by the given line so as to contain an angle equal to a given angle”; (c) [pages 498–500] dated 1662, “To inscribe a regular heptagon in a circle”; (d) [pages 521–523] dated 1666, “To find the integer which, when divided by three given integers, the remainders are three given integers.”

Problem (a)—After algebraic analysis Huygens gives a resulting geometrical construction and proof. An earlier and different discussion, by synthetic geometry, was published in 1607 by M. Ghetaldi in his *Variorum Problematum Collectio*, Venice, 1607, pp. 25–27; see also Oughtred, *Clavis Mathematicae*, third edition, Oxford, 1652, pp. 86–87. The more general problem, “Given the vertical angle, the sum of the two including sides, and the difference of the segments of the base made by a perpendicular dropped from the vertex, to construct the triangle,” was solved by Renaldini in his *De Resolutione et Compositione Mathematica*, Padua, 1668, pp. 319, 529, and by T. Simpson, in his *A Treatise of Algebra*, London, 1745, pp. 292–293.

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Problem (b)—This problem was proposed to Huygens by R. F. de Sluse, in a letter dated ¹ October 23, 1657, and Huygens here shows merely how the solution of the problem may be reduced to the solution of a quadratic equation.

In a letter to Fermat, dated July 29, 1654, Pascal refers² to two problems

¹ *Oeuvres Complètes de Christian Huygens*, vol. 2, 1889, pp. 72 and 80.

² *Oeuvres de Fermat*, vol. 2, Paris, 1894, p. 298.